

Determination of the strong g -coupling and of the $\mu_{B^*B\gamma}$ — strength from two-photon decays of heavy vector mesons

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We describe a calculation of the two-photon decays of heavy vector-mesons, $B^* \rightarrow B\gamma\gamma$ and $D^* \rightarrow D\gamma\gamma$, by using the “heavy meson chiral Lagrangian”. The decay amplitudes are expressed in terms of the strong coupling g of the Lagrangian at various powers and the strength of the anomalous magnetic dipole $\mu_{B^*B\gamma}$ (respectively $\mu_{D^*D\gamma}$). In the charm case we are able to express the branching ratio $\Gamma(D^{*0} \rightarrow D^0\gamma\gamma)/\Gamma(D^{*0})$ as a function of g only and we expect it to be in the $10^{-6} - 10^{-5}$ range, depending on the value of g . The determination of $\mu_{B^{*0}B^0\gamma}, \mu_{B^{*+}B^+\gamma}$ requires a more involved analysis, including the consideration of bremsstrahlung radiation for the $B^{*+} \rightarrow B^+\gamma\gamma$ case.

1. INTRODUCTION

The heavy vector mesons D^* and B^* decay by strong and electromagnetic interactions. Since the mass difference $M_{B^*} - M_B$ is only 45.78 ± 0.35 MeV [1], the decay $B^* \rightarrow B\pi$ cannot occur and the main decay of B^* is $B^* \rightarrow B\gamma$. On the other hand, the $D^* - D$ mass difference barely allows the decay to a pion and accordingly the main decays are $D^* \rightarrow D\pi, D\gamma$. The strength of the couplings involved in these processes, $g_{D^*D\pi}, g_{D^*D\gamma}, g_{B^*B\gamma}$ for the different charge states is not known although relative branching ratios for the various $D^* \rightarrow D\pi, D^* \rightarrow D\gamma$ isotopic spin states are well measured [1].

In recent years, a theoretical framework has been developed for the description of the low energy strong interactions [2,3] and of the electromagnetic interactions [3,4] between mesons containing a heavy quark Q and the pseudoscalar Goldstone bosons. The Heavy Quark Chiral Lagrangian (HQ χ L) developed for this framework [2,3] combines the flavour and spin symmetry of the heavy quark effective theory with the $SU(3)_L \otimes SU(3)_R$ chiral symmetry of the light sector, allowing the use of chiral perturbation theory for treating the interaction of heavy

mesons with low-momentum pions. The symmetry of this Lagrangian leads to defining a strong coupling g which represents the four couplings $g_{B^*B\pi}, g_{B^*B^*\pi}, g_{D^*D^*\pi}, g_{D^*D\pi}$ (as well as couplings to η , etc) and a coupling μ which represents the anomalous magnetic couplings $g_{B^*B^*\gamma}, g_{B^*B\gamma}, g_{D^*D^*\gamma}, g_{D^*D\gamma}$. This is true in the symmetry limit and corrections will obviously appear when one considers deviations from both the heavy quark limit and the zero-mass chiral limit.

At present, there is no experimental determination of the strength of any of these couplings. On the other hand a variety of theoretical models has been advanced for their calculation (see [5] for a recent review). As a result, there is a rather wide range of theoretical possibilities for g and μ , since the predictions of various models differ considerably. In the next section we shall present a short survey of this situation. One might expect a direct determination of $g_{D^*D\pi}$ from the measurement of the decay width of $D^* \rightarrow D\pi$; presently, there is only an upper limit of $\Gamma_{\text{tot}}(D^{*+}) < 131\text{keV}$ [6] and the possibility of an actual measurement depends on the magnitude of the physical value. It is therefore of obvious interest to devise independent methods for the determination of g and μ couplings. We shall

describe here our recent suggestion [7,8] of using the $B^* \rightarrow B\gamma\gamma$ and $D^* \rightarrow D\gamma\gamma$ decays for the determination of g and $g_{B^*B\gamma}$. Using the Heavy Quark Chiral Lagrangian we shall express the amplitudes for $B^* \rightarrow B\gamma\gamma$, $D^* \rightarrow D\gamma\gamma$ in terms of g and μ . A detailed analysis will show that the D^* decay is particularly useful for the determination of g while the $B^{*+} \rightarrow B^+\gamma\gamma$ decay may possibly be used to determine the strength of $g_{B^{*+}B^+\gamma}$.

2. THE THEORETICAL FRAMEWORK

The decays we are considering, which were mentioned firstly in [7], are $B^{*+} \rightarrow B^+\gamma\gamma$, $B^{*0} \rightarrow B^0\gamma\gamma$, $B_s^{*0} \rightarrow B_s^0\gamma\gamma$ and $D^{*+} \rightarrow D^+\gamma\gamma$, $D^{*0} \rightarrow D^0\gamma\gamma$, $D_s^{*+} \rightarrow D_s^+\gamma\gamma$. We shall use for their description the HQ χ L in the leading order of chiral perturbation theory. Corrections to it requires terms proportional to the mass of the light quarks m_q , to $(1/M_Q)$ and derivatives in the HQ χ Lagrangian [9-11]; this question will be commented on in the last section.

The interaction term of HM χ L to lowest order is [2,3]

$$\mathcal{L}^{\text{int}} = g \text{Tr} \{ \bar{H}_a \gamma_\mu \gamma_5 A_{ab}^\mu H_b \} \quad (1)$$

which defines the basic strong-interaction coupling g . H_a is a 4×4 Dirac matrix, with one spinor index for the heavy quark Q and the other for the light quark q ,

$$H = \frac{1 + \not{v}}{2} [B_\mu^* \gamma^\mu - B \gamma_5], \quad \bar{H} = \gamma_0 H^\dagger \gamma_0 \quad (2)$$

and $B_{a\mu}^*(v)$, $B_a(v)$ are the velocity-dependent annihilation operators of the meson fields, with $v^\mu B_{a\mu}^* = 0$. a, b denote light quark flavours ($a, b = 1, 2, 3$) and the B_μ^* , B fields represent, unless otherwise specified, both the beauty and charm sectors. A_{ab}^μ is the axial current containing an odd number of pion fields, given by $A_\mu = \frac{1}{2}(\zeta^\dagger \partial_\mu \zeta - \zeta \partial_\mu \zeta^\dagger)$, $\zeta = \exp(i\mathcal{M}/f)$ with \mathcal{M} being the 3×3 matrix of the octet of pseudoscalar Nambu-Goldstone bosons. f is the pion decay constant, $f = 132$ MeV. Expanding A_μ and keeping the first term $A^\mu = (i/f)\partial_\mu \mathcal{M}$ we obtain the explicit interaction terms from (1)

$$\begin{aligned} \mathcal{L}_{(1)}^{\text{int}} = & \left[-\frac{2g}{f} B_\mu^* \partial^\mu \mathcal{M} B^\dagger + H.c. \right] \\ & + \frac{2gi}{f} \epsilon_{\mu\nu\sigma\tau} B^{*\mu} \partial^\sigma \mathcal{M} B^{*\dagger\nu} v^\tau. \end{aligned} \quad (3)$$

With the usual definition for the strong $B^*B\pi$, $B^*B^*\pi$ [5] vertices

$$\langle \pi(q) \bar{B}(v_1) | B^*(v_2, \epsilon_2) \rangle = g_{B^*B\pi}(q^2) q_\mu \epsilon_2^\mu \quad (4)$$

$$\begin{aligned} & \langle \pi(q) \bar{B}^*(v_1, \epsilon_1) | B^*(v_2, \epsilon_2) \rangle \\ & = g_{B^*B^*\pi}(q^2) \epsilon_{\alpha\beta\mu\nu} \epsilon_1^\alpha \epsilon_2^\beta q^\mu v_1^\nu \end{aligned} \quad (5)$$

the physical couplings are given by the limit $q^2 \rightarrow m_\pi^2$. Isospin symmetry is expressed by the relations

$$\begin{aligned} g_{B^*B\pi} & \equiv g_{B^{*+}B^0\pi^+} = -\sqrt{2}g_{B^{*+}B^+\pi^0} = \\ & \sqrt{2}g_{B^{*+}B^0\pi^0} = -g_{B^{*0}B^+\pi^-} \end{aligned} \quad (6)$$

and similarly for the $g_{B^*B^*\pi}$ couplings. Comparing (3) to (4), (5) we obtain these couplings expressed in terms of g , and at this point we distinguish between beauty and charm

$$g_{B^*B\pi} = g_{B^*B^*\pi} = (2M_B/f)g, \quad (7.1)$$

$$g_{D^*D\pi} = g_{D^*D^*\pi} = (2M_D/f)g. \quad (7.2)$$

Throughout this work, we follow the normalization conventions of [5]. In (7.1), (7.2) mass degeneracy for (B^*, B) and (D^*, D) pairs was assumed and the use of the pseudoscalar mass value in these equations is conventional.

The incorporation of electromagnetism in HM χ L is performed by the usual procedure of minimal coupling, which leads to the replacement of derivatives in the free Lagrangian \mathcal{L}_0 by covariant derivatives containing the $U(1)$ photon field, when

$$\begin{aligned} \mathcal{L}_0 = & -i \text{Tr} \{ \bar{H}_a v^\mu D_{\mu ba} H_b \} + \\ & \frac{f^2}{8} \text{Tr} \partial_\mu \Sigma \partial^\mu \Sigma^\dagger. \end{aligned} \quad (8)$$

Here $\Sigma = \zeta^2(X)$ and $D_\mu = \partial_\mu + V_\mu$ with the vector current, containing an even number of pions, given by $V_\mu = \frac{1}{2}(\zeta^\dagger \partial_\mu \zeta + \zeta \partial_\mu \zeta^\dagger)$

The interaction vertices of the heavy mesons with photons are then [3].

$$\langle \gamma(k, \epsilon) \bar{B}(v_1) | B(v_2) \rangle = e M_B (v_1 + v_2) \cdot \epsilon \quad (9.1)$$

$$\begin{aligned} & \langle \gamma(k, \epsilon) \bar{B}^*(v_1, \epsilon_1) | B^*(v_2, \epsilon_2) \rangle = \\ & e M_{B^*} (\epsilon_1 \cdot \epsilon_2) (v_1 + v_2) \cdot \epsilon. \end{aligned} \quad (9.2)$$

These vertices are not capable to provide for the existing $B^*B\gamma$ interaction and an additional gauge invariant term proportional to the electro magnetic field $F_{\mu\nu}$ must be added [3,4,11] to the Lagrangian $\mathcal{L}_0 + \mathcal{L}^{\text{int}}$. It is given as

$$\mathcal{L}^{(\mu)} = \frac{e\mu}{4} \text{Tr}\{\bar{H}_a \sigma_{\mu\nu} F^{\mu\nu} H_b \delta_{ab}\} \quad (10)$$

which gives when we express H_a from Eq. (2)

$$\mathcal{L}^{(\mu)} = -e\mu F^{\mu\nu} [iB_\mu^{*+} B_\nu^* + \epsilon_{\mu\nu\sigma\tau} v^\sigma (B^+ B^{*\tau} + H.c.)] \quad (11)$$

This translates into two ‘‘anomalous’’ vertices for the electromagnetic interaction, in addition to (91.), (9.2),

$$\langle \gamma(k, \epsilon) \bar{B}^*(v_1, \epsilon_1) | B^*(v_2, \epsilon_2) \rangle =$$

$$e\mu M_{B^*}(\epsilon_1 \cdot k\epsilon \cdot \epsilon_2 - \epsilon_2 \cdot k\epsilon \cdot \epsilon_1), \quad (12.1)$$

$$\langle \gamma(k, \epsilon) \bar{B}(v_1) | B^*(v_2, \epsilon_2) \rangle =$$

$$-ie\mu M_{B^*} \epsilon_{\mu\nu\sigma\tau} \epsilon^\mu k^\nu v_2^\sigma \epsilon_2^\tau. \quad (12.2)$$

The new coupling μ is related to the customary physical couplings by

$$M_{B^*}\mu^0 = g_{B^*0B^0\gamma}; \quad M_{D^*}\mu^0 = g_{D^*0D^0\gamma}, \quad (13.1)$$

$$M_{B^*}\mu^+ = g_{B^{*+}B^+\gamma}; \quad M_{D^*}\mu^+ = g_{D^{*+}D^+\gamma}. \quad (13.2)$$

The values of $g_{B^*B\pi}$, $g_{D^*D\pi}$, $g_{B^*B^*\pi}$, $g_{D^*D^*\pi}$, $g_{B^*B\gamma}$, $g_{D^*D\gamma}$, $g_{B^*B^*\gamma}$, $g_{D^*D^*\gamma}$ are not known from direct experiments; however, a vast literature of theoretical attempts has appeared and we expose now succinctly the emerging theoretical picture concerning g and μ and the respective physical couplings expressed in Eqs. (7.1),(7.2),(13.1),(13.2). It should be stressed here that the wide interest in g is due also to the fact that its strength appears in the expressions for many low-energy electroweak processes, like $B \rightarrow \pi\ell\nu$, $D_s \rightarrow K\ell\nu$, $B \rightarrow D^*\pi\ell\nu$ among others (see [5] for comprehensive review).

One may group roughly the calculations of g into four major classes, those based on constituent quark models, those using relativistic quark models, the use of QCD sum rules and the effective Lagrangian approach. There are obviously different approaches within the same group as well as calculations which cannot be classified as mentioned. Most calculations obtain, in fact,

the values of $g_{B^*B\pi}$ and/or $g_{D^*D\pi}$. The naive quark model result is $g = 1$ [3] and its modification by the inclusion of chirality [12] brings it slightly down to $g \simeq 0.75 - 0.8$. The use of relativistic quark approaches [13], of which recently a new wave of results has appeared gives values of g of approximately 0.45-0.65. The QCD sum-rules tend to give [14,15] lower values, $g \simeq 0.2 - 0.4$. We also mention a lattice QCD calculation [16] with the result $g = 0.42$ (4), the analysis of Stewart [11] which uses HQ χ L with corrections to order m_q and $1/M_Q$ and constraints from the D^* decay branching ratio to obtain $g = 0.27^{+0.09}_{-0.04}$, and a chiral bag model calculation [17] which has predicted accurately the observed D^* branching ratio and gives $g = 0.53$. We should caution the reader on the use in the literature of different definitions for the couplings involved. Thus $g_{D^*D\pi}$ defined in [15] is larger by the factor (M_{D^*}/M_D) than the one used, e.g. in Refs. [5,7,12]. Moreover, in certain works one defines a coupling \hat{g} (see [15]), which is related to g of Eqs. (7.1), (7.2) by $\hat{g} = (M_{D^*}/M_D) \left(1 + \frac{\Delta}{M_D}\right)^{-1} g$, where Δ is $1/M_Q$ correction with a value of 0.7 ± 0.1 .

Experimentally [6], there is the upper limit of $\Gamma(D^{*+} \rightarrow D^0\pi^+) < 89\text{KeV}$, which translates into $g < 0.71$. The prediction of the naive quark model for this mode is 120-180 KeV, of the relativistic quark models 55-80 KeV, of the QCD sum rules 10-30 KeV, of Ref.[17] is 53 ± 3 KeV and of Ref. [11] is 18 KeV.

The situation concerning μ is very similar. Generally, the same models employed for calculating g were used also for getting the $D^* \rightarrow D\gamma$, $B^* \rightarrow B\gamma$ modes. Since the relative branching ratios for $D^{*+,0}$ decays into $D\pi$ vs. $D^{+,0}\gamma$ are known [1], most calculations insist on reproducing these ratios; then the calculated absolute partial widths for $D^* \rightarrow D\gamma$ decays follow the same pattern as the calculations of g , i.e. predicting rather small width from QCD sum rules [14,18] and larger ones from quark models [13], chiral approaches [4,17] or potential models [19]. The range of variation for the different predictions is about one order of magnitude in rate. Parallel predictions are made for the $B^{*0} \rightarrow B^0\gamma$, $B^{*+} \rightarrow B^+\gamma$ decays. To exemplify with typical results, the QCD sum-rule approach of Dosch and Narrison and of [18] predicts $\Gamma(B^{*0} \rightarrow B^0\gamma) =$

0.04KeV, $\Gamma(B^{*+} \rightarrow B^+\gamma) = 0.1\text{KeV}$, while a chiral bag model calculation [20] obtains $\Gamma(B^{*0} \rightarrow B^0\gamma) = 0.28\text{KeV}$, $\Gamma(B^{*+} \rightarrow B^+\gamma) = 0.62\text{KeV}$ and even higher values were obtained in certain models ([4], [9] and first ref. of [13]).

3. TWO PHOTON DECAY AMPLITUDES

We use now HM χ L with electromagnetic interactions, as detailed in the previous section, to calculate the $B^* \rightarrow B\gamma\gamma$ process (B stands for both beauty and charm). We begin by treating the neutral decays only, which are free from bremsstrahlung. The calculation is performed to leading order in chiral perturbation theory and to this order there are no unknown counterterms [11,21]. The decay amplitude is given at this order by [7,8]

$$A^0 = A_{\text{anomaly}}^0 + A_{\text{tree}}^0 + \sum_{i=1}^6 A_{\text{loop}}^{0(i)}. \quad (14)$$

We shall describe now these various contributions to the decay amplitude stressing the couplings, without giving the involved detailed expressions which can be found in [7].

A_{anomaly}^0 represents $B^{*0} \rightarrow B^0 \pi^0 \rightarrow B^0 \gamma\gamma$ via a virtual pion. In the charm case, where the decay to a physical pion is allowed, we limit the $s = (k_1 + k_2)^2$ variable to be up to 20 MeV away from the pion mass. This contribution is proportional to αg , with $\alpha = e^2/4\pi$. A_{tree}^0 is given by the transition $B^{*0} \rightarrow B^{*0}\gamma \rightarrow B\gamma\gamma$, having two insertions of μ . Hence A_{tree}^0 is proportional to $\alpha\mu_0^2$. Then we have six classes of diagrams containing pions and/or kaons loop, and the photon emission may occur from loop or from the external legs via the anomalous μ interaction. We shall describe a typical diagram for each of these six classes. $A_{\text{loop}}^{0(1)}$ describes $B^{*0} \rightarrow (B^{*+}\pi^-) \rightarrow B^0\gamma\gamma$ with the two photons radiated from the virtual charged pion and is proportional to αg^2 . The radiation from the virtual B^{*+} is negligible. $A_{\text{loop}}^{0(2)}$ represents the transition $B^{*0} \rightarrow B^{*0}\gamma \rightarrow (B^{*+}\pi^-)\gamma \rightarrow B^0\gamma\gamma$ and is proportional to $\alpha\mu_0 g^2$. $A_{\text{loop}}^{0(3)}$ is a similar class, with the $B^{*0}B^0\gamma$ vertex replacing the $B^{*0}B^{*0}\gamma$ one in the first step, thus being also proportional to $\alpha\mu_0 g^2$. $A_{\text{loop}}^{0(4)}$ describes the tran-

sition $B^{*0} \rightarrow (B^{*0}\pi^0)\gamma \rightarrow B^{*0}\gamma \rightarrow B^0\gamma\gamma$ being proportional to $\alpha g^2\mu_0$. Replacing B^* by B in the loop one gets $A_{\text{loop}}^{0(5)}$ which is also proportional to $\alpha g^2\mu_0$. Finally, we have $B^{*0} \rightarrow (B^{*+}\pi^-)\gamma\gamma \rightarrow B^0\gamma\gamma$; this describes double radiation by the loop, with one photon radiated by the pion and the other one due to the anomalous transition μ . This amplitude is proportional to $\alpha\mu_+ g^2$.

Calculating the rate of the decay one obtains an expression containing 13 terms which depend on the products $g^\alpha \mu_0^\beta \mu_+^\gamma$ with different powers, α and β having values between 0 and 4 while γ has values between 0 and 2.

Turning to the $B^{*+} \rightarrow B^+\gamma\gamma$ amplitude, we must consider now also the bremsstrahlung given by vertices (9.1), (9.2). Again, we just mention here the diagrams involved, explicit expressions being given in [22]. Firstly, there are all the diagrams classified as in (14), except that μ_0 and μ_+ are interchanged. However, the major diagrams now are those involving bremsstrahlung radiation. Let us denote the additional diagrams by $B^{(b)} = B_{\text{tree}}^{(b)} + B_{\text{loop}}^{(b)}$. $B_{\text{tree}}^{(b)}$ has two classes of contributions: (1) the chain $B^{*+} \rightarrow B^{*+}\gamma \rightarrow B^+\gamma\gamma$ where the first vertex has strength e and the second $e\mu_+$; (ii) the chain $B^{*+} \rightarrow B^{*+}\gamma \rightarrow B^+\gamma\gamma$ where the first vertex is $e\mu_+$ and the second is e . $B_{\text{loop}}^{(b)}$ has similarly a contribution from $B^{*+} \rightarrow \gamma B^{*+} \rightarrow \gamma \pi^+ B^{*0} \rightarrow \gamma\gamma B^+$ and from $B^{*+} \rightarrow \pi^+ B^{*0} \rightarrow \gamma B^+ \rightarrow \gamma\gamma B^+$. The radiation from the external legs is due to the (9.1), (9.2) vertices.

In performing the calculations we used for the propagator of the vector meson [5] $-i(g^{\mu\nu} - v^\mu v^\nu)/2[(v \cdot k) - \Delta/4]$ and for that of the scalar meson $i/2[(v \cdot k) + 3\Delta/4]$, where $\Delta = M_{B^*} - M_B$. We also employed physical masses in diagram calculations and rates, thus including some features of the $1/M_Q$ corrections; moreover, the chiral loops we include are themselves of order $1/M_Q$.

4. ANALYSIS AND CONCLUSIONS

The analysis of the various modes involves different features and must be carried out separately. We start with the neutral decay $D^{*0} \rightarrow D^0\gamma\gamma$. For this case, the crucial step is to use the experimental data on the relative branching ratios of the strong and radiative D^{*+}, D^{*0} decays

$\Gamma(D^{*0} \rightarrow D^0\pi^0) : \Gamma(D^{*0} \rightarrow D^0\gamma) = (61.9 \pm 2.9)\% : (38.1 \pm 2.9)\%$ and $\Gamma(D^{*+} \rightarrow D^0\pi^+) : \Gamma(D^{*+} \rightarrow D^+\gamma) = (67.7 \pm 0.5)\% : (1.6 \pm 0.4)\%$ [1]. This allows us to relate μ_0 and μ_+ to g and to obtain an expression for $\Gamma(D^{*0} \rightarrow D^0\gamma\gamma)$ in terms of g at various powers only [6]

$$\Gamma(D^{*0} \rightarrow D^0\gamma\gamma) = [2.52 \times 10^{-11}g^2 + 5.66 \times 10^{-11}g^3 + 4.76 \times 10^{-9}g^4 + 3.64 \times 10^{-10}g^5 + 1.53 \times 10^{-9}g^6] \text{ GeV}. \quad (15)$$

For $\Gamma(D^{*0}) = \Gamma(D^{*0} \rightarrow D^0\gamma) + \Gamma(D^{*0} \rightarrow D^0\pi^0)$ we use the same procedure for the relating of μ_0 to g and one obtains $\Gamma(D^{*0} \rightarrow \text{all}) = (2.02 \pm 0.12) \times 10^{-4}g^2 \text{ GeV}$. This leads to our main result concerning the measurement of the value of g : we have shown that the measurement of the branching ratio

$$Br(D^{*0} \rightarrow D^0\gamma\gamma) =$$

$$\Gamma(D^{*0} \rightarrow D^0\gamma\gamma) / (\Gamma(D^{*0} \rightarrow D^0\gamma) + \Gamma(D^{*0} \rightarrow D^0\pi^0))$$

given by

$$Br(D^{*0} \rightarrow D^0\gamma\gamma) = \frac{0.0025 + 0.057g + 4.76g^2 + 0.36g^3 + 1.53g^4}{2.02 \times 10^5} \quad (16)$$

is a direct measurement of the strong coupling g . In the expressions (15), (16) we assumed relative positive signs for g, μ_0, μ_+ as indicated by theory [11]. However, even if we assume negative relative signs for various pairs of the three parameters g, μ_0, μ_+ , the changes are very small [6] since the main contribution is coming from quadratic terms.

Since g is expected to be in the range $0.25 < g < 1$, our result (16) indicates that the branching ratio is expected to be $0.17 \times 10^{-5} < Br(D^{*0} \rightarrow D^0\gamma\gamma) < 3.3 \times 10^{-5}$. The absolute width of the two-photon decay obtains from (15) to be $2.2 \times 10^{-2} \text{ eV} < \Gamma(D^{*0} \rightarrow D^0\gamma\gamma) < 6.7 \text{ eV}$ for g in the 0.25-1 range.

It is of interest to remark here that for $0.1 < g < 0.7$ the main contribution to the decay rate of $D^{*0} \rightarrow D^0\gamma\gamma$ is given by the g^2, g^3, g^4 terms of (15). Out of these terms, the more important one is the g^4 one, which accounts for more than 90% of the contribution for $g > 0.25$. The g^4 term gets contributions from the tree term, from the $A_{\text{loop}}^{0(1)}$ term and from the interference of the

anomaly with the $A_{\text{loop}}^{02} - A_{\text{loop}}^{05}$ terms. However, the first one is by far the major contribution, accounting for about 90% of this term. Hence, the tree term emerges as the major contributor to the two photon decay $D^{*0} \rightarrow D^0\gamma\gamma$ for g in the “preferred” region $0.2 < g < 0.7$. For smaller values the anomaly becomes competitive, while for g closer to one, the loop terms play this role. The differential decay width $d\Gamma(D^{*0} \rightarrow D^0\gamma\gamma)/ds$ is given in [6] for $g = 0.25$ and $g = 0.7$ and the role of the anomaly is visible in the first graph.

The situation for $B^{*0} \rightarrow B^0\gamma\gamma$, which amplitude depends also on μ_0, μ_+ and g , is considerably more difficult to analyze since the help of measured relative rates which we had for D^* decays is unavailable here. The branching ratio to the main decay $B^{*0} \rightarrow B^0\gamma$ thus remains dependent on the three unknown couplings in this case. Nevertheless, some help comes from the fact that the influence of the μ_+ is very small and the process may therefore be analyzed in the reduced parameter space of $\{\mu_0, g\}$. As shown in [6], the dependence on the assumed relative sign of g/μ_0 is however not negligible now. Moreover, the differential decay width does not vary enough with the variation of μ_0, g , in order to provide for a reliable determination. It appears that the best approach is to wait for the determination of g from D^* decays. Then the measurement of $B^{*0} \rightarrow B^0\gamma$, both differential and total rate, will provide a measurement of μ_0 , — for which no other alternative exists. Concerning the expected branching ratio, for $0.25 < g < 0.7$ and $40 \text{ eV} < \Gamma(B^{*0} \rightarrow B^0\gamma) < 1 \text{ KeV}$, $Br\{\Gamma(B^{*0} \rightarrow B^0\gamma\gamma)/\Gamma(B^{*0} \rightarrow B^0\gamma)\}$ varies between 3.1×10^{-7} and 1.5×10^{-5} .

Lastly, we refer to the charged decays, $D^{*+} \rightarrow D^+\gamma\gamma$, $D_s^{*+} \rightarrow D_s^+\gamma\gamma$, $B^{*+} \rightarrow B^+\gamma\gamma$ and let us take the latter as an example. Now, the expected rate is much larger than in the neutral modes, since bremsstrahlung radiation is involved. To get an idea of the effect, just taking diagram $B_{\text{tree}}^{(b)II}$ described in the previous section, one finds a branching ratio $Br(B^{*+} \rightarrow B^+\gamma\gamma/B^{*+} \rightarrow B^+\gamma)$ of 0.7×10^{-2} for $k_1, k_2 > 10 \text{ MeV}$. This branching ratio will be mainly a function of g and μ_+ . In [22] a detailed analysis is presented of the effect of varying the strength of these couplings on the branching ratio and differential spectrum of

$B^{*+} \rightarrow B^+ \gamma \gamma$. Again, the knowledge of g will simplify the problem considerably.

In concluding, we remark that our calculation [6] was performed to the leading order in chiral perturbation theory and mostly to leading order in $1/M_Q$. Our purpose was to introduce the method, to show how one could measure g from $D^{*0} \rightarrow D^0 \gamma \gamma$ decay and to propose a possible measurement of $\mu_{B^{*+}B^+\gamma}, \mu_{B^{*0}B^0\gamma}$ from the analysis of $B^* \rightarrow B \gamma \gamma$ decays. Corrections to the leading order should be performed, along the methods developed in recent years [5,11,23], but these will not affect the qualitative features of the method we proposed.

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